

Review question R9585

Question

The point lying between $P(2, 3)$ and $Q(8, -3)$ which divides the line PQ in the ratio $1 : 2$ has co-ordinates

- (a) $(4, -1)$
- (b) $(6, -2)$
- (c) $(\frac{14}{3}, 2)$
- (d) $(4, 1)$

Oxford University Mathematics Aptitude Test, Specimen 2, Q1A

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Solution

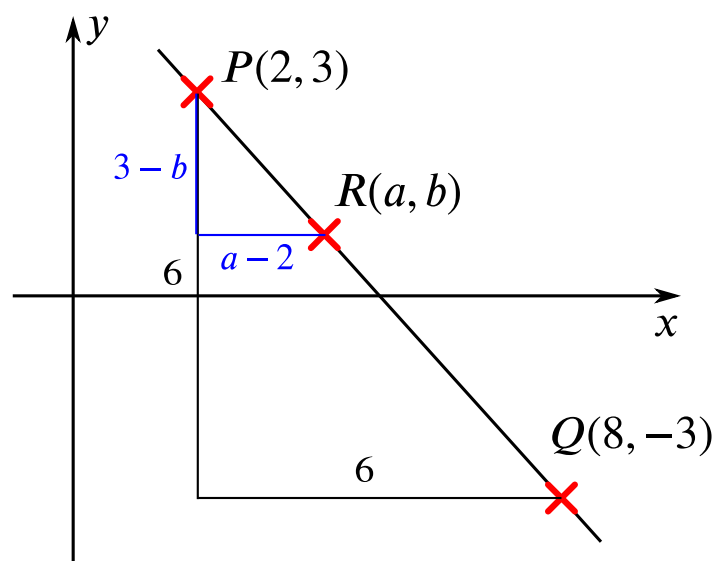


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Approach 1: Similar triangles

We sketch the situation:



Let $R = (a, b)$ divide the line PQ in the ratio $1 : 2$ with $PR : RQ = 1 : 2$, so R lies one third of the way along PQ .

By similar triangles, $\frac{PR}{PQ} = \frac{1}{3} = \frac{a-2}{6} = \frac{3-b}{6}$, which gives $a = 4$, $b = 1$.

Approach 2: Vectors

We can also answer this question using vectors.

$$\text{Now } \vec{OP} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \text{ while } \vec{OQ} = \begin{pmatrix} 8 \\ -3 \end{pmatrix}.$$

$$\text{Hence } \vec{PQ} = \vec{OQ} - \vec{OP} = \begin{pmatrix} 6 \\ -6 \end{pmatrix}.$$

$$\text{Now } \vec{OR} = \vec{OP} + \frac{1}{3}\vec{PQ} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}.$$

Either way, we reach the answer d.

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